

Deterministic creation and stabilization of entanglement in circuit QED by homodyne-mediated feedback control

Zhuo Liu, Lülin Kuang, Kai Hu, Luting Xu, Suhua Wei, Lingzhen Guo * and Xin-Qi Li †
Department of Physics, Beijing Normal University, Beijing 100875, China

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In the solid-state circuit QED system and based on the homodyne measurement in dispersive regime, we demonstrate that a homodyne-current-based feedback can create and stabilize highly entangled two-qubit states in the presence of moderate noisy environment. Particularly, we present an extended analysis for the current-based Markovian feedback, which leads to an improved filtered-current-based feedback scheme. We show that this is essential for us to achieve the desirable control effect in present system.

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I. INTRODUCTION

The circuit quantum electrodynamics (QED) [1–3], a solid-state analog of the conventional quantum optics cavity QED, is a promising solid-state quantum computing architecture. This architecture couples superconducting electronic circuit elements, which serve as the qubits, to harmonic oscillator modes of a microwave resonator, which serve as a “quantum bus” that mediates inter-qubit coupling and facilitates quantum measurement for the qubit state. At the early stage of experiments, strong coupling between the qubits and the quantum bus in the circuit QED system should make it excellent platform for quantum control study.

In particular, quantum measurement in this system can be carried out by operating in the dispersive limit (where the detuning between the resonator and the qubit is much larger than their coupling strength). In this limit, the interaction induces a qubit state dependent frequency shift on the resonator. By measuring the resonator output voltage with a homodyne measurement, information about the qubit state is obtained. In our present work, based on this homodyne measurement together with the flexibility/advantage that feedback can be applied to either the qubit or the microwave resonator mode, we will illustrate how highly entangled state can be created and stabilized from an initially separable state, by an appropriate feedback control. It has been well known that quantum entanglement is one of the key ingredients in order to realize quantum computation and quantum information processing. Very recently, in the circuit QED system, interesting ideas were proposed to *probabilistically* create entangled states by means of the homodyne measurement alone [4], following the general idea that measurement can be used as a nondeterministic means of preparing quantum states that are otherwise difficult to obtain. However, besides the drawback of probabilistic nature, this *measurement alone* approach has no ability to stabilize/protect the obtained entangled state. Generally speaking, entanglement degradation through uncontrolled coupling with the environment remains a major obstacle in practice [5–

7], which would limit the lifetime of entangled states and demand efficient schemes to protect them. In this context we remind that, owing to the remarkable progress in theory and particularly in experiments on the real-time monitoring and manipulation of individual quantum systems [8], the quantum feedback control technique may emerge as a natural possible route to develop strategies to prepare entangled states and prevent their deterioration.

In Ref. [9], for the optical cavity QED system, the steady-state of two qubits (two-level atoms) interacting simultaneously with a driving laser was shown to be entangled, achieving a concurrence of 0.11 without measurement or feedback. A later work, involving the use of a homodyne-mediated *direct* feedback modulation of the laser that drives the cavity mode [10], extended this study and showed that one can increase the steady-state entanglement, i.e., the amount of the maximum steady-state concurrence can be increased from 0.11 to 0.31. In principle, maximally entangled states could be achieved [11], by the use of Bayesian, or state estimation, feedback [12]. This improvement, however, comes at the price of increasing the experimental complexity, due to the challenging need for a real-time estimation of the quantum state. More recently, rather than the homodyne-mediated, a jump-based direct feedback is demonstrated to allow the robust production of highly entangled states in the optical cavity system [13]. For the solid-state circuit QED, however, the single photon counting at microwave frequencies is currently not possible, despite a couple of proposals and efforts out there for doing it [14, 15]

In this paper we base the feedback on the more available homodyne measurement scheme [16]. Particularly, in Ref. [17], detailed analysis for measurement scheme to reach the quantum limit was presented, and a deterministic creation of entanglement by using the measurement based feedback was illustrated in the absence of environmental decoherence. In present work we extend the study in Ref. [17] by taking into account the environmental influence. We will show that all the four Bell states can be deterministically created and stabilized in the presence of moderate noisy environment, with high quality compared to the existing results mentioned above.

*E-mail: guolingzhen@mail.bnu.edu.cn

†E-mail: lixinqi@bnu.edu.cn

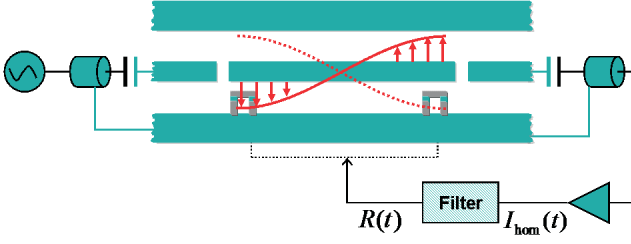


FIG. 1: Schematic diagram of circuit QED together with a microwave transmission measurement and a measurement-current-based feedback loop. The Cooper-pair box qubits are fabricated inside a superconducting transmission-line resonator and are capacitively coupled to the voltage standing wave.

II. MODEL AND FORMALISM

In Fig. 1 we illustrate the schematic setup of the solid-state superconducting circuit-QED system, together with the measurement and feedback control idea. The central section of superconducting coplanar waveguide plays the role of the cavity and the superconducting qubits play the roles of the atoms. The superconducting qubits are coupled to a one-dimensional transmission line (1DTL) cavity which acts as a simple harmonic oscillator. The qubits, the 1DTL cavity and their mutual coupling can be well described by the Jaynes-Cummings Hamiltonian:

$$H = \omega_r a^\dagger a + \mathcal{E}(a^\dagger + a) + \sum_{j=1,2} \left[\frac{\Omega_j}{2} \sigma_j^z + g_j (\sigma_j^- a^\dagger + \sigma_j^+ a) \right]. \quad (1)$$

Here, the operators $\sigma_j^- (\sigma_j^+)$ and $a (a^\dagger)$ are, respectively, the lowering (raising) operators for the j th qubit and the cavity photons. ω_r is the frequency of the cavity photon, and Ω_j and g_j are the j th qubit transition energy and coupling strength to the cavity photon. For simplicity and for the purpose to be clear later, in this work we assume that $\Omega_1 = \Omega_2 = \Omega$, and $g_1 = -g_2 = g$. This implies that we assume two identical qubits located in the cavity at places with the maximum field amplitude and with a half-wavelength separation, as schematically shown in Fig. 1. The \mathcal{E} -term in Eq. (1) stands for a microwave driving to the cavity that is employed here for the task of measurement. More explicitly, $\mathcal{E} = \epsilon e^{-i\omega_m t} + \text{c.c.}$, where the frequency can differ from the cavity photon frequency, i.e., $\Delta_r \equiv \omega_r - \omega_m \neq 0$. In concern with the qubit-cavity coupling, we focus on the dispersive regime [1–3], which corresponds to an energy detuning, $\Delta = \omega_r - \Omega$, much larger than g . In this limit, the canonical transformation, $H_{\text{eff}} \simeq U^\dagger H U$, where $U = \exp[\sum_j \lambda_j (a \sigma_j^+ - a^\dagger \sigma_j^-)]$ with $\lambda_j = g_j / \Delta$, yields (in the rotating frame with the driving frequency ω_m)

$$H_{\text{eff}} \simeq \Delta_r a^\dagger a + \epsilon(a + a^\dagger) + (\Omega + \chi) J_z / 2 + \chi a^\dagger a J_z + \chi (\sigma_1^+ \sigma_2^- + \sigma_1^- \sigma_2^+). \quad (2)$$

Here, we defined $\chi = g^2 / \Delta$ and $J_z = \sigma_1^z + \sigma_2^z$.

In the circuit-QED system, the measurement is implemented via a homodyne-type detection of the transmitted microwave photons as schematically shown in Fig. 1. For the

composite system of the qubits plus the 1DTL cavity, the leakage of photons is described by a Lindblad term $\kappa \mathcal{D}[a] \rho$ in master equation, where κ is the leakage rate and the Lindblad superoperator acting on the reduced density matrix ρ is defined by $\mathcal{D}[a] \rho = a \rho a^\dagger - \frac{1}{2} \{a^\dagger a, \rho\}$. However, conditioned on the output (homodyne) current, i.e., $I_{\text{hom}}(t) = \kappa \langle a + a^\dagger \rangle_c(t) + \sqrt{\kappa} \xi(t)$, there will be an additional unravelling term in the conditional master equation, $\mathcal{H}[a] \rho_c \xi(t)$. Here, $\langle (\cdots) \rangle_c(t) \equiv \text{Tr}[(\cdots) \rho_c(t)]$ with $\rho_c(t)$ the conditional density matrix, and $\mathcal{H}[a] \rho_c \equiv a \rho_c + \rho_c a^\dagger - \text{Tr}[(a + a^\dagger) \rho_c] \rho_c$. And, the quantum-jump related stochastic nature is characterized by $\xi(t)$, a Gaussian white noise with properties of $E[\xi(t)] = 0$ and $E[\xi(t) \xi(t')] = \delta(t - t')$, where $E[\cdots]$ means an ensemble average over realizations of the noise. Furthermore, in this work, we are interested in the regime of strongly damped cavity, which enables us to adiabatically eliminate the cavity degree of freedom [18]. Qualitatively, from an observation on the effective coupling $\chi a^\dagger a J_z$ in Eq. (2), we can expect: the measurement-backaction induced dephasing term $\sim \mathcal{D}[J_z] \rho_c$, the unravelling term $\sim \mathcal{H}[J_z] \rho_c \xi(t)$, and the homodyne current $I_{\text{hom}}(t) \sim \langle J_z \rangle_c(t) + \xi(t)$. Indeed, following the standard procedures of adiabatic elimination [18], one obtains the quantum trajectory equation (QTE) involving only the qubit degree of freedom, in a rotating frame with respect to the qubit Hamiltonian which reads

$$\dot{\rho}_c = -i\chi |a|^2 [J_z, \rho_c] + \sum_{j=1,2} \gamma_j \mathcal{D}[\sigma_j^-] \rho_c + \sum_{j=1,2} \frac{\gamma_{\phi j}}{2} \mathcal{D}[\sigma_j^z] \rho_c + \gamma_p \mathcal{D}[\sigma_1^- - \sigma_2^-] \rho_c + \frac{\Gamma_d}{2} \mathcal{D}[J_z] \rho_c + \frac{\sqrt{\Gamma_m}}{2} \mathcal{H}[J_z] \rho_c \xi(t). \quad (3)$$

In deriving this result, we have used the assumption $\Delta_r = 0$ and $\lambda_1 = -\lambda_2 = \lambda$. In Eq. (3), γ_j and $\gamma_{\phi j}$ are the relaxation and dephasing rates caused by the surrounding environment, while $\gamma_p = \kappa \lambda^2$ is a rate for the collective decay due to the Purcell effect. Explicitly, the measurement-backaction induced dephasing rate $\Gamma_d = 8|a|^2 \chi^2 / \kappa$, with $\alpha = -2i\epsilon / \kappa$. Finally, the information gain rate Γ_m in Eq. (3) is in general related to the backaction dephasing rate through the quantum efficiency, $\eta = \Gamma_m / (2\Gamma_d)$.

III. HOMODYNE-MEDIATED FEEDBACK

A. Feedback Equation in Markovian Limit

Based on the QTE, one can infer the conditional state $\rho_c(t)$. In principle, one can then perform a state-estimate based feedback, by an appropriate design for the feedback Hamiltonian. This is the so-called Bayesian feedback scheme. Another, much simpler, scheme is the direct feedback which depends linearly on the detection signal, e.g., the homodyne current in the present case. Indeed, the former scheme can result in an improvement over the direct feedback, yet it comes at the cost of an increasing experimental complexity due to the challenging need for a real-time estimation of the quantum state. In what follows, we focus on the direct current-based feedback scheme. In general, we denote the feedback Hamiltonian as

$H_{fb}(t) = I_{hom}(t - \tau)\hat{F}$, where τ stands for a possible time delay of the feedback, while the homodyne current can be expressed as $I_{hom}(t) = \sqrt{\Gamma_m}\langle J_z \rangle_c(t) + \xi(t)$ after the adiabatic elimination of the cavity degree of freedom. Performing this feedback leads to state change according to $[\dot{\rho}_c(t)]_{fb} = I_{hom}(t - \tau)\mathcal{K}\rho_c(t)$, where $\mathcal{K}\rho_c(t) \equiv -i[\hat{F}, \rho_c(t)]$. In the Markovian limit by assuming $\tau = 0$, an appropriate combination of this equation with the QTE (3) yields

$$\begin{aligned} \dot{\rho}_c = & -i\chi|\alpha|^2[J_z, \rho_c] + \frac{\Gamma_d}{2}\mathcal{D}[J_z]\rho_c \\ & + \sum_{j=1,2} \gamma_j \mathcal{D}[\sigma_j^-]\rho_c + \sum_{j=1,2} \frac{\gamma_{\phi j}}{2} \mathcal{D}[\sigma_j^z]\rho_c \\ & + \gamma_p \mathcal{D}[\sigma_1^- - \sigma_2^-]\rho_c + \frac{\sqrt{\Gamma_m}}{2} \mathcal{K}(J_z \rho_c + \rho_c J_z) \\ & + \frac{1}{2} \mathcal{K}^2 \rho_c + \left(\frac{\sqrt{\Gamma_m}}{2} \mathcal{H}[J_z] + \mathcal{K} \right) \rho_c \xi(t). \end{aligned} \quad (4)$$

Originally, this type of equation was obtained by Wiseman and Milburn [19], by a careful interpretation to the feedback as an Ito- or Stratonovich-type stochastic action. Interestingly, we mention here an alternate (and equivalent) derivation for this result. Since even in the Markovian limit the feedback can be applied only after the measurement outcome, in terms of discretized time interval we thus have $\rho_c(t + dt) = e^{d\mathcal{K}(t)}\tilde{\rho}_c(t + dt)$, where $\tilde{\rho}_c(t + dt)$ is the state after inferring from the measurement record, and the differential superoperator reads $d\mathcal{K}(t) = dI_{hom}(t)\mathcal{K}$. Here, $dI_{hom}(t) = \sqrt{\Gamma_m}\langle J_z \rangle_c(t)dt + dW(t)$, while $dW(t) = \xi(t)dt$ is the Wiener increment that has the statistical properties of $E[dW(t)] = 0$ and $E[dW(t)dW(s)] = \delta(t - s)dt$. Expanding this instantly fast feedback to second order and keeping the infinitesimal increment to $O(dt)$ (noting that $[dW(t)]^2 = dt$), one then obtains Eq. (4).

B. Entanglement Creation and Stabilization: Preliminary Result

From now on we specify our study by using feedback to create two-qubit entanglement, first based on Eq. (4), then on an improved scheme. In particular, we will focus on feedback protection against the spontaneous emission of qubits, while assuming the dephasing rates $\gamma_{\phi j}$ negligibly small for the transmon-type qubit [4]. For two qubits, there are four maximally entangled states, i.e., the Bell states: $|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, and $|\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$. In what follows we only detail our analysis for $|\Phi_{+}\rangle$, and remain the others as brief discussion in the final concluding section. Initially, we assume a separable state for the two qubits, $(|0\rangle + |1\rangle)_1 \otimes (|0\rangle + |1\rangle)_2 = (|00\rangle + |11\rangle) + (|10\rangle + |01\rangle)$, which can be prepared easily by separate single bit rotations. Within the scheme after adiabatic elimination of the cavity degree of freedom, we see that the current $\langle J_z \rangle_c$ is zero for qubit state $|10\rangle + |01\rangle$, and nonzero otherwise. Then, we expect that the current-based feedback would force the state towards the entangled state $|10\rangle + |01\rangle$, if we exert a J_x -type flipping, i.e., apply a feed-

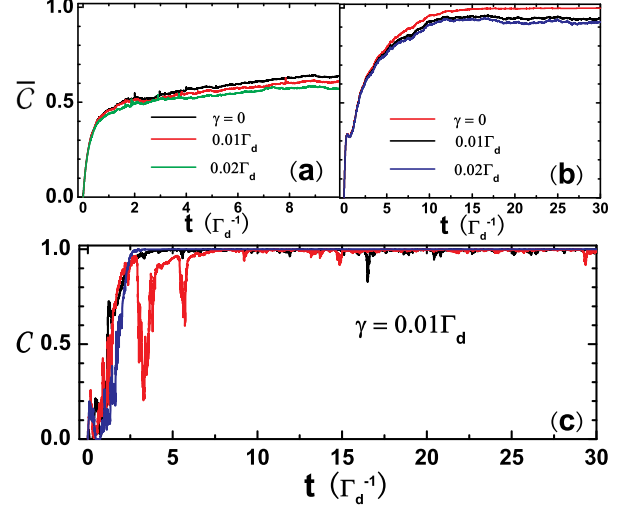


FIG. 2: (a): Average concurrence over 500 trajectories based on Eq. (4) and a direct current feedback $H_{fb} = u I_{hom}(t) J_x$. (b) and (c): Concurrence obtained by a feedback $H_{fb} = u \langle J_z \rangle_c(t) J_x$, i.e., removing the noise in the homodyne current. In (b) the average and in (c) the individual quantum trajectory concurrences are illustrated, both showing considerable improvement over the result in (a). Parameters: $\gamma_p = \Gamma_d$, $u = 0.1$ in (a) and $u = 1.0$ in (b) and (c). Here, the choice of different feedback strength $u = 0.1$ in (a) is from a rough numerical optimization, since an increase of u over this value will give poorer result instead.

back $H_{fb}(t) = u I_{hom}(t) J_x$, where u characterizes the feedback strength over the current-based modulation.

In Fig. 2(a), we illustrate the control result based on Eq. (4), where the average concurrence over 500 trajectories is shown. Note that the concurrence, which is particularly useful to characterize two-qubit entanglement in a mixed state[20], is an effective measure in present context for the creation and stabilization of $|\Phi_{+}\rangle$, which is in good consistence with the state fidelity, $F(t) = \text{Tr}[|\Phi_{+}\rangle\langle\Phi_{+}|\rho(t)]$. Compared to Refs. [9] and [10], where the achieved concurrences are, respectively, 0.11 and 0.31, we see that the result in Fig. 2(a) is good, but not optimal for the following reasons. First, we note that this improvement over the result in Ref. [10] is largely owing to the different measurement scheme here, which in the dispersive regime is in fact a continuous quantum non-demolition (QND) measurement. That is, here the measurement is a J_z -type, while in Ref. [10] it is a J_x -type. For current-based feedback, as heuristically discussed above, the homodyne current $dI_{hom}(t)$ contains, not only the useful signal $\langle J_z \rangle_c(t)$, but also the harmful noise (i.e. the $dW(t)$ term). To this point, it is important to understand the distinct role of the $dW(t)$ term in the homodyne current, when using it to update the quantum state *versus* to perform the feedback. In doing the former, it is informative; while in doing the latter, it is useless and harmful. In the current-based Markovian feedback equation, i.e., Eq. (4), the two Wiener increments were equated and were mixed up

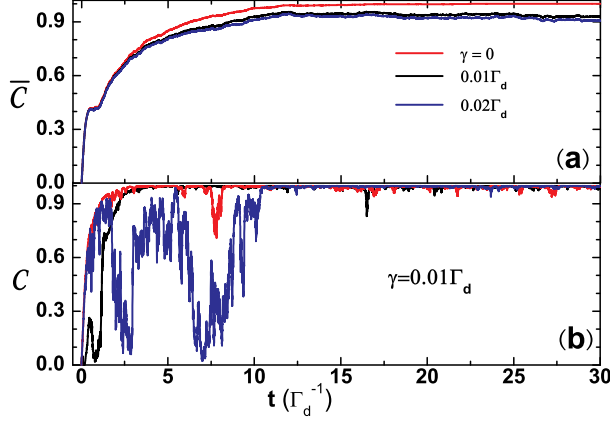


FIG. 3: (a) Average concurrence over 500 trajectories, and (b) the concurrence of three representative individual trajectories, by applying a filtered-current-based feedback control. Parameters: $\gamma_p = \Gamma_d$, $u = 10$, $\gamma_{\text{fit}} = 0.006\Gamma_d$, and $T = 2000 * dt$.

in the final form [19]. We thus understand that it may lead to inefficient result sometimes. As a quantitative support to this reasoning, in Fig. 2(b) and (c), we show the result by adding a feedback, $H_{\text{fb}} = u\langle J_z \rangle_c(t)J_x$, straightforwardly into the measurement record conditioned quantum trajectory equation. We remind here that the noise in the homodyne current was removed. In Fig. 2(b), the average concurrence by averaging over 500 quantum trajectories is plotted, while in Fig. 2(c) the concurrence of individual quantum trajectories is illustrated. Noticeably, essential improvement over the result in Fig. 2(a) is achieved.

C. Filtered-Current-Based Feedback: Improved Result

As a matter of fact, the above “improved” feedback, $H_{\text{fb}} = u\langle J_z \rangle_c(t)J_x$, is a state-estimation feedback, since the $\langle J_z \rangle_c(t)$ is known *only* after knowing the state $\rho_c(t)$. However, guided by it, we can refine the noisy homodyne current. Following Ref. [17], we first low-pass filter the measurement signal over a small time window, $R(t) = \frac{1}{N} \int_{t-T}^t e^{-\gamma_{\text{fit}}(t-\tau)} dI_{\text{hom}}(\tau)$, where the factor N normalizes the maximum of the smoothed signal $R(t)$ to unity. Through this filtering procedure, we get in fact a crude but efficient estimate of $\langle J_z \rangle_c(t)$. Then, we condition the feedback with a power of the filtered measurement signal, which would reduce the noise further in the estimate. That is, we finally design a feedback Hamiltonian, $H_{\text{fb}}(t) = uR(t)^P J_x$, adding directly into the measurement conditioned quantum trajectory equation, where P is the power to which the smoothed signal is raised. In Fig. 3 we display the (a) average and (b) individual trajectory concurrences, based on this filtered-current feedback control. We find that the result is satisfactory, being comparable to that in Fig. 2(b) and (c).

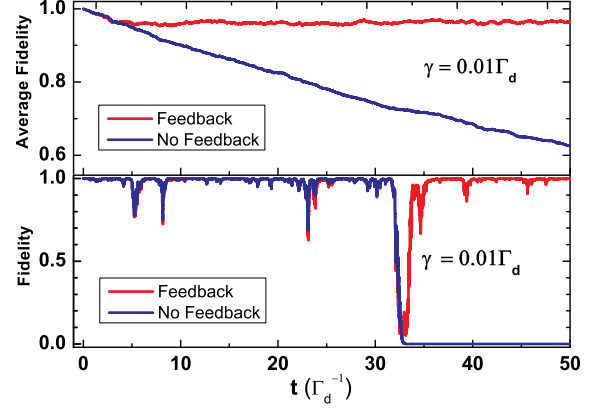


FIG. 4: Environment caused deterioration of the Bell state $|\Phi_+\rangle$ in the absence of feedback protection. In (a) the average fidelity (over 1000 trajectories) and in (b) the fidelity of a representative trajectory are presented (blue curves), while the result with feedback is plotted here for comparison (red curves). The feedback parameters are the same as in Fig. 3.

IV. ENTANGLEMENT DETERIORATION IN THE ABSENCE OF FEEDBACK

In addition to the *deterministic* creation of entanglement achieved here, we remark that the other advantage of feedback is its ability to *stabilize* the entanglement. We illustrate this point more explicitly by the following. Given the entangled state $|\Phi_+\rangle$ having been achieved, for instance, probabilistically by the *measurement alone* approach as proposed in Ref. [4], in Fig. 4 we plot the later fate of this state in terms of its fidelity $F(t) = \text{Tr}[(|\Phi_+\rangle\langle\Phi_+|)\rho(t)]$, in the absence of protection with feedback. In Fig. 4(a) we show the average fidelity for the ensemble averaged state, which is found to decay with time under the influence of environment. By altering the coupling strength to environment, this decay may turn to the remarkable phenomena of sudden death of entanglement [5], which were discussed under a broad class of dissipation models. Meanwhile, in Fig. 4(b), the fidelity of representative individual trajectory is shown. Interestingly, the sudden jump of fidelity to zero is observed, which indicates a sudden disappearance of entanglement in our present case. We stress that this is a real “sudden death” of entanglement, in the sense of an instantaneous disappearance of entanglement, in contrast to the gradual death (i.e., with finite lifetime) of the ensemble averaged state [5]. Anyhow, we may conclude, that even the entangled state (e.g. $|\Phi_+\rangle$) has been probabilistically created [4], it will completely die, probably after some short surviving time, but almost certainly with the increase of time, in the absence of feedback protection. Nevertheless, the feedback we discussed above can prevent this from happening.

V. DISCUSSION AND SUMMARY

So far we have examined a feedback scheme of deterministic creation and stabilization of entanglement. As a specific example, we considered the state $|\Phi_+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$ in detail. In practice, the need of “ $J_x (= \sigma_1^x + \sigma_2^x)$ ”-type feedback can be realized by a modulation of the qubit Hamiltonian parameters, as schematically indicated in Fig. 1, or more simply, by modulating the microwave driving strength. For the latter scheme, besides the measurement driving, one can apply an additional control microwave driving, with amplitude of ϵ_c and frequency in resonance with the qubit transition energy. It can be shown that this driving will induce an effective $H_{\text{dr}} = \lambda \epsilon_c J_x$ on the qubits, if we properly set the qubits so that $g_1 = g_2 = g$, or equivalently, $\lambda_1 = \lambda_2 = \lambda$. Notice that in this case the Purcell term turns out to be $\gamma_p \mathcal{D}[\sigma_1 + \sigma_2]\rho$, instead of $\gamma_p \mathcal{D}[\sigma_1 - \sigma_2]\rho$ as in Eq. (3), making the state $|\Phi_+\rangle$ not invariant under its action. However, one can make this term relatively small by appropriately increasing the detuning between the cavity photon and the qubit. Moreover, since this term only transits $|\Phi_+\rangle$ to $|00\rangle$, yet $J_x|00\rangle = |\Phi_+\rangle$, we thus expect that the applied feedback can well suppress its destructive effect.

Using similar method, one can deterministically create and stabilize another entangled state $|\Phi_-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$. This can be realized by a homodyne-mediated \tilde{J}_x -type feedback, where $\tilde{J}_x \equiv \sigma_1^x - \sigma_2^x$. This feedback can still be implemented by modulating either the qubit Hamiltonian parameters or the microwave driving. Finally, in concern with the other two Bell states, $|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$, we can create and stabilize them by a simple conversion scheme, by noting that they are in fact related to $|\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$ by a single bit π -pulse σ_x rotation. Note that direct creation and stabilization of $|\Psi_{\pm}\rangle$ is also possible, by alternatively setting $g_1 = g_2 = g$ and an opposite detuning $\Omega_1 - \omega_r = \omega_r - \Omega_2$. Furthermore, an extension to protect arbitrary entangled states, say, $|\tilde{\Phi}_{\pm}\rangle = c|01\rangle \pm d|10\rangle$ and $|\tilde{\Psi}_{\pm}\rangle = c|00\rangle \pm d|11\rangle$, is straightforward, by simply replacing the feedback operator J_x by an appropriate combination of σ_1^x and σ_2^x .

Finally, we mention that the main problem with doing feedback in circuit QED is the lack of efficient homodyne detection. Currently, the way to perform homodyne and heterodyne detection is to first amplify the signal before mixing it on a nonlinear circuit element of some kind. As a consequence, the extra noise added by the amplifier will reduce the quantum efficiency and prohibit quantum limited feedback. It seems that this situation is to be changed quickly, for instance, by developing Josephson parametric amplifiers which can be realized in superconducting circuits [16]. In our present theoretical study, we did not include the non-unit quantum efficiency into

the homodyne detection of the field. This treatment is largely owing to the following consideration. After adiabatic elimination of the cavity photon degree of freedom, the non-unit quantum efficiency of homodyne detection will reduce the effective information-gain rate Γ_m in Eq. (3). This implies an emergence of an extra non-unravelling dephasing term in the quantum trajectory equation. However, in the context of creation and stabilization of $|\Phi_+\rangle$, this term only results in dephasing among states $|00\rangle$, $|11\rangle$, and $|01\rangle + |10\rangle$. That is, it does not destroy the target state $|\Phi_+\rangle$. From the feedback principle discussed in Sec. III (B), we can also imagine that this dephasing term only affects the detailed control dynamics, but does not prevent the state towards the target state. Therefore, for certain acceptable (not too low) quantum efficiency of the homodyne detection, the present scheme of feedback should be implementable in circuit QED. This is seemingly a rare example that the control effect does not sensitively depend on the measurement efficiency. We have numerically examined the above reasoning, for instance, by lowering the quantum efficiency to $\eta = 0.8$, and found very small change of the entanglement concurrence.

To summarize, in the solid-state circuit QED system and based on the homodyne measurement in dispersive regime, we analyzed the creation and stabilization of the highly entangled Bell states by the use of a filtered-current-based feedback. Compared to a few previous studies in similar cavity QED systems, we (i) eliminate the drawback of probabilistic nature and the inability of stabilization [4], (ii) improve the control effect by enhancing the concurrence to values higher than 0.9, in regard to Ref. [10] where the improved concurrence of 0.31 over the former 0.11 was achieved, and (iii) avoid the experimental difficulty in present system for the jump-based feedback [13], or the complexity for the state-estimation feedback [11].

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